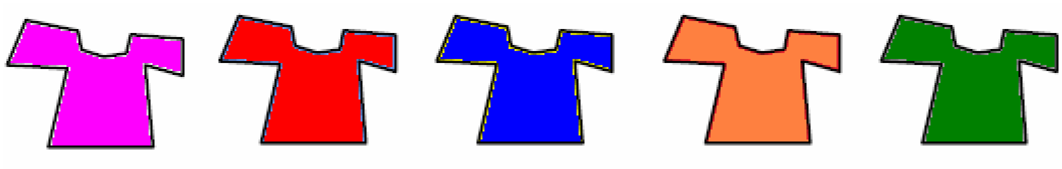
1. ***If you have five different colored shirts and are going to pack two of them to go on a weekend trip, how many possibilities are there for the two that you select?***

Solution:

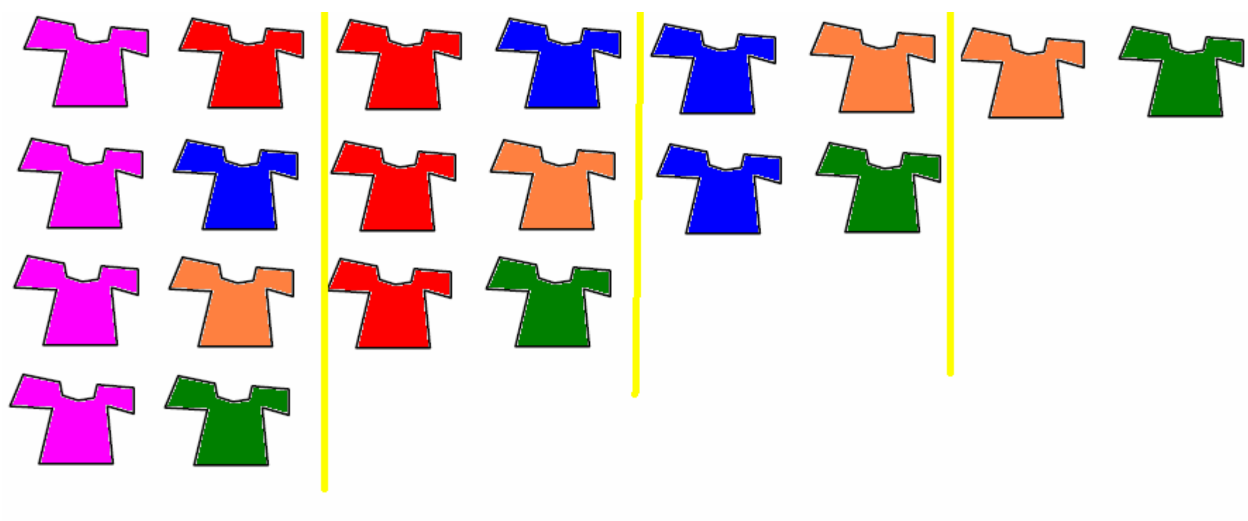
Let's assume we have 5 different colored shirts in the wardrobe



Let us pick 2 shirts among them.

We can pick them with any combinations

Let us arrange all the various types of combination



So we can clearly see that we have 10 possibilities to pick 2 shirts out of 5shirts

Since we have less number of shirts (5) we have drawn the figures and got all the combinations

If they are large number of shirts, can we drawn figures for all the shirts and also for all combinations?

Answer is **simply no or it's very tedious and lengthy**

So how do we do this mathematically?

By using the concept of Permutations and Combinations, we can solve them easily

Since we have 5 shirts and we have to pick 2 shirts among them

So the number of combinations is $C(5,2) = \frac{5!}{2!3!}$

$$C(5,2) = \frac{5*4*3*2*1}{(2*1)(3*2*1)} = 10$$

2. Solve 2sin*x* - 1 = 0

Solution:

We first look at **sin*x*** as being the variable of the equation and solve for it,

Like $2\sin x - 1 = 0$

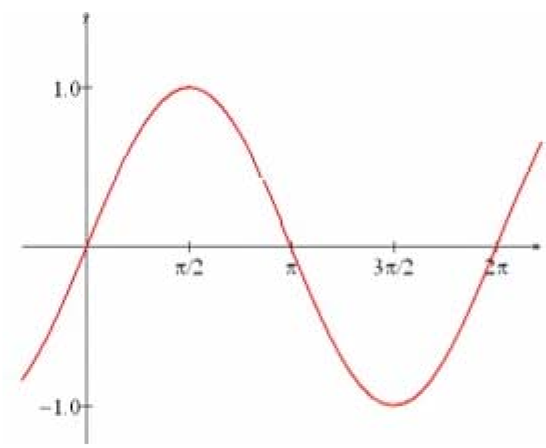
$2\sin x = 1$

$\sin x = ½$ ----(i)

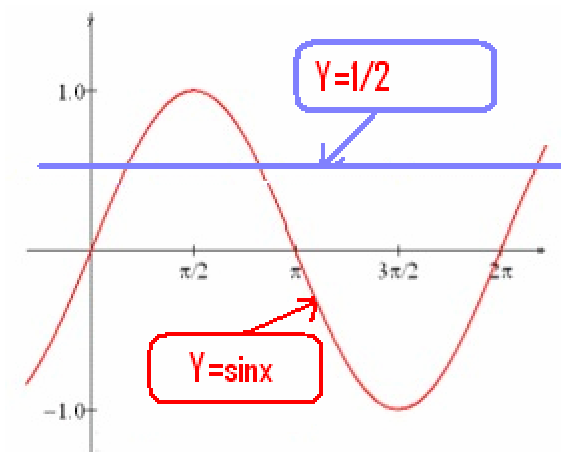
Now we have to find the values of x which satisfies the equation (i)

Let's recall the graph of sinθ

It varies from 0 to 2π

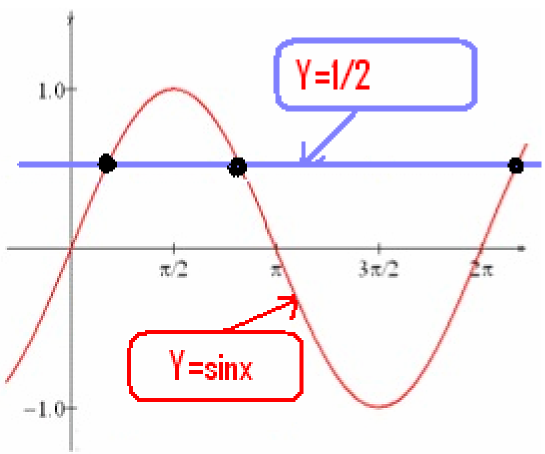


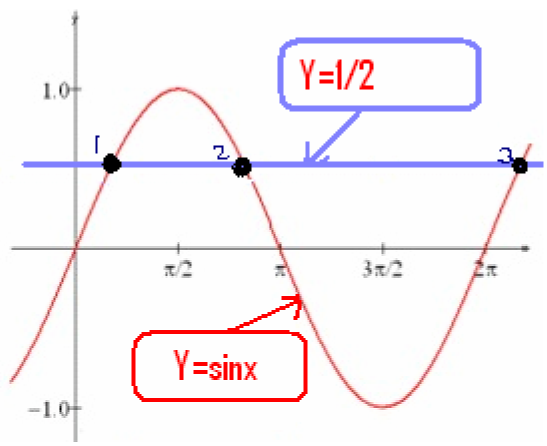
Now Lets draw the line which satisfies y=1/2 on the graph of sinθ



So we can clearly notice that the line Y=1/2 touches the curve Y=sinx at 3 points

Let mark the points





In the range of [0,2π], we can clearly have the points 1 and 2

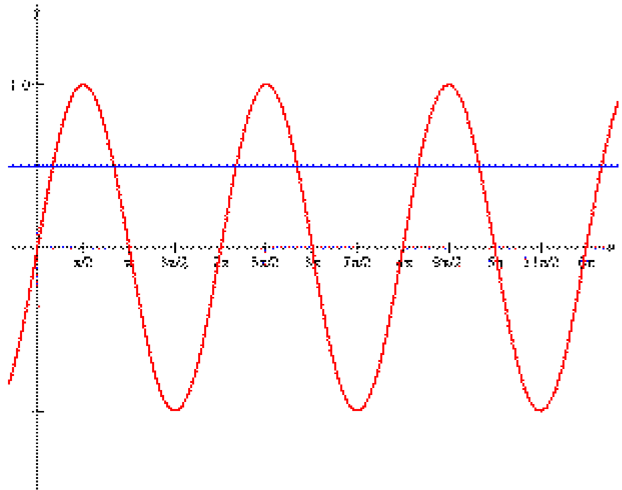
So these are the two points which satisfies the equation y=sinx

These values are at:

$$\theta = \frac{\pi}{6} \quad \text{and} \quad \theta = \frac{5\pi}{6}$$

or at 30º and 150º

If we look at the extended graph of $\sin\theta$, we see that there are many other solutions to this equation $\sin\theta$ = 1/2.



We can easily find all other solutions by adding a multiple of 2π to θ.

$\frac{\pi}{6} + 2n\pi \;\; and \;\; \frac{5\pi}{6} + 2n\pi$ where n is an integer in $[0,\infty)$.

3. ***Evaluate this series*** : $\sum_{k=1}^{14}(1-2k)$

Solution:

Here the Symbol ∑ stands for Summation

So it means that we have sum up all the terms of (1-2k) where the values of k are 1,2,3…14

So let's expand the summation

First term (k=1) : 1-2k = 1-2*1=-1

Second term(k=2) : 1-2k = 1-2*2=-3

third term(k=3) : 1-2k = 1-2*3=-5

Let's add all the terms

-1 + (-3) + (-5) + ... + (-27)

We can clearly see that it is following an Arithmetic sequence

With Initial Term (a) = -1 and Common Difference (d) = -2

The sum of a certain number of terms of an **arithmetic sequence is:**

$$S_n = \frac{n(a_1 + a_n)}{2}$$

where S_n is the sum of n terms (**n^{th} partial sum**), a_1 is the first term, a_n is the n^{th} term

as 'k' is from 1,2,3…14, so there are 14 terms(n=14)

$$S_{14} = \frac{14(-1+(-27))}{2} = -196$$

4. ***A survey was taken in biology class regarding the number of siblings of each student. The table shows the class data with the frequency of responses. The mean of this data is 2.5. Find the value of k in the table.***

Siblings (x)	1	2	3	4	5
Frequency (f)	5	***k***	8	4	1

Solution:

First of all, let's see

What is mean?

It is the value found by adding product of data times the corresponding frequencies in the data set and dividing by the sum of frequencies in that set.

Mean X is defined as

$$\bar{x} = \frac{X1f1 + X2f2 + X3f3 + \ldots\ldots + Xnfn}{f1 + f2 + f3 \ldots\ldots\ldots\ldots\ldots + fn}$$

Now let's plug in the values to find the Mean

$$\bar{x} = \frac{(1)(5) + (2)(k) + (3)(8) + (4)(4) + (5)(1)}{5 + k + 8 + 4 + 1}$$

$$\bar{x} = \frac{50 + 2k}{18 + k}$$

But mean of this data is given as 2.5

$$\therefore \quad \bar{x} = \frac{50+2k}{18+k} = 2.5$$

$\Rightarrow 50+2k = (2.5)(18+k)$

$\Rightarrow 50+2k = 45+2.5k$

$\Rightarrow 5 = 0.5k$

$\Rightarrow k = 10$

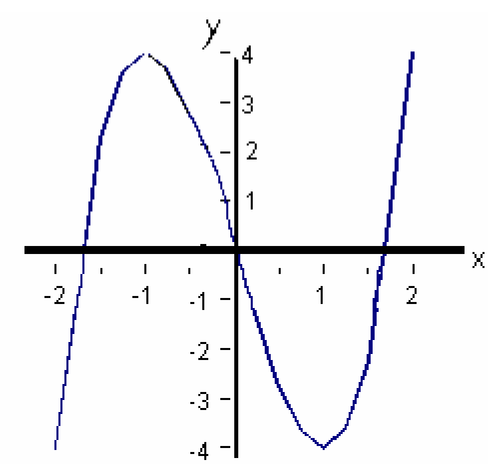
5. What is the area under the curve $y(x) = 2x^3 - 6x$ between $x = -1$ and $x = 0$?

Solution:

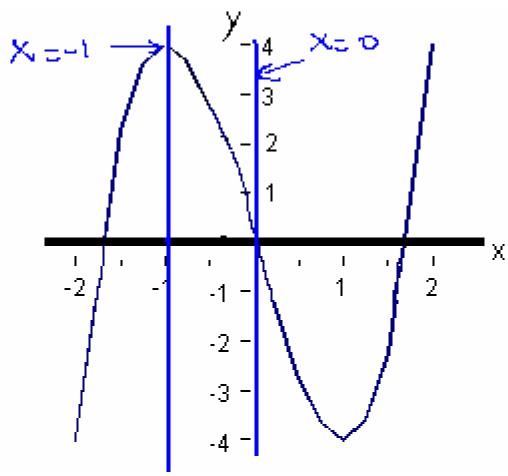
Given curve is $y(x)=2x^3-6x$

Area to be find in the range x=-1 and x=0?

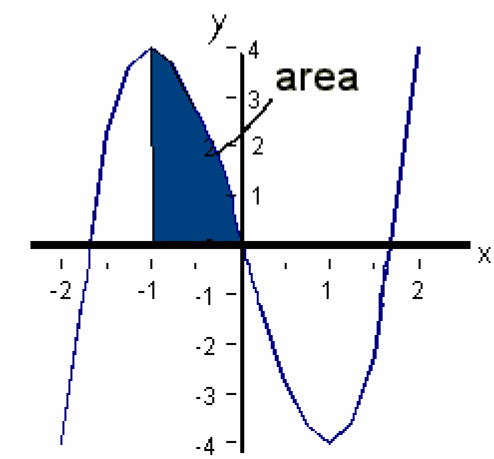
First of all lets plot the curve $y(x)=2x^3-6x$ on the graph



Now lets draw the line x = -1 and x = 0 on the same graph



Now Lets shade the region under the curve y(x) = $2x^3$ - 6x between x = -1 and x = 0



So the Shaded area gives the area under the curve. So the Area will be the number of units which is 2 ½ sq. units.

Let's do this Mathematically,

We can do these using definite integrals

$$A = \int_{x=-1}^{x=0} 2x^3 - 6x.dx$$

$$= \left[\frac{x^4}{2} - 3x^2\right]_{-1}^{0}$$

Now by applying the limits

A = (0-0) - (½ - 3) = **<u>2½ sq.units</u>**

Hence Area under the under the curve $y(x) = 2x^3 - 6x$ between $x = -1$ and $x = 0$ is **2½ sq.units**